

B.Sc. Part I, Paper

Linear Programming

Convex Set :- Let  $S$  be a vector space over the real numbers or over some ordered field. A subset  $C$  of  $S$  is convex if, for all  $x$  and  $y$  in  $C$ , the line segment connecting  $x$  and  $y$  is included in  $C$ . This means that the affine combination  $(1-t)x + ty$  belongs to  $C$ , for all  $x$  and  $y$  in  $C$ , and  $t$  in the interval  $[0,1]$ . This implies that convexity is invariant under affine transformations. This means a convex set in a real or complex topological vector space is path-connected, thus connected.

A set  $C$  is strictly convex if every point on the line segment connecting  $x$  and  $y$  other than the endpoints is inside the interior of  $C$ . A set  $C$  is absolutely convex if it is convex and balanced.

The convex subsets of  $\mathbb{R}$  (the set of real numbers) are the intervals and the points of  $\mathbb{R}$ .

Some examples of convex subsets are solid regular polygons, solid triangles, and intersections of solid triangles etc.

Some examples of convex subsets of a Euclidean 3-dimensional space are the Archimedean solids and the Platonic solids.

The Kepler-Poinsot polyhedra are the example of non-convex sets.

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Non-convex Set:- A set that is not convex is called a non-convex set. A polygon that is not a convex polygon is sometimes called a concave polygon, and some sources more generally use the term concave set to mean a non-convex set, ~~but most~~ ~~with~~ the complement of a convex set, such as the epigraph of a concave function, is sometimes called a reverse convex set, especially in the context of mathematical optimization.

### Properties of convex set:

Given  $r$  points  $u_1, \dots, u_r$  in a convex set  $S$ , and  $r$  nonnegative numbers  $\lambda_1, \dots, \lambda_r$  such that  $\lambda_1 + \dots + \lambda_r = 1$ , the affine combination  $\sum_{k=1}^r \lambda_k u_k$  belongs to  $S$ . As the definition of a convex set is the case  $r=2$ , this property characterizes convex sets. Such an affine combination is called a convex combination of  $u_1, \dots, u_r$ .

- 1) The empty set and the whole space are convex.
- 2) The intersection of any collection of convex sets is convex.
- 3) The union of a sequence of convex sets is convex, if they form a non-decreasing chain for inclusion. For this property, the restriction to chains is important, as the union of two convex sets need not be convex.